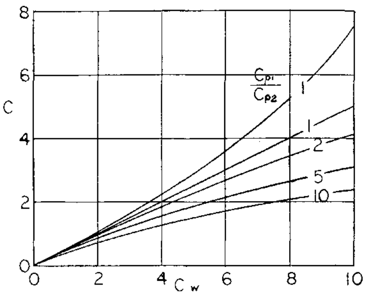

Fig 2 Concentration at boundary layer maximum temperature, $h_\infty/h_{0\infty} \ll 1$, $h_w/h_{0\infty} \ll 1$, $h_{0\infty} = h_\infty + u_\infty^2/2g \simeq u_\infty^2/2g$, $Pr = Le = 1$, (1) → injected fluid, (2) → original fluid

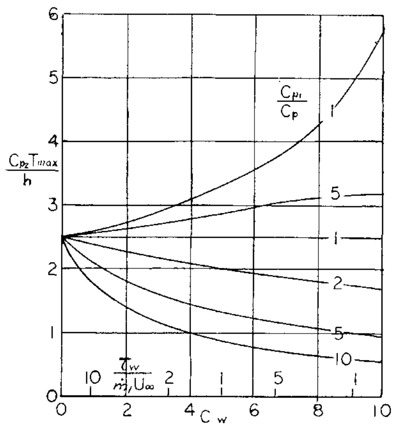

Fig 3 Maximum boundary layer temperature with injection, $h_\infty/h_{0\infty} \ll 1$, $h_w/h_{0\infty} \ll 1$, $h_{0\infty} = h_\infty + u_\infty^2/2g \simeq u_\infty^2/2g$, $Pr = Le = 1$, (1) → injected fluid, (2) → original fluid

ation of $\rho\mu$ has the same influence on injection as on skin friction under the simplifying assumptions, and Eq (18) is independent of the gas being injected and the wall temperature The solution in Ref 3, being simply that of Chapman and Rubesin with a normal velocity component at the wall, does not consider any interaction between the injected and original gases, thus implying that transport properties are similar In Refs 1 and 4, the authors have considered boundary layers in which there is a large variation of $\rho\mu$ A large number of gases viscosities are within a factor of 2 or so; therefore, $\rho\mu$ varies within an order of magnitude as density or molecular weight For an effective average value of C' the "reference enthalpy" method has been used, and, at least for the stagnation point solutions, the following relation has proved adequate:

$$\langle c' \rangle = (\rho_w\mu_w/\rho_w\mu_\infty)^{0.2} \qquad (20)$$

In Ref 5 numerical results of air/air, CO_2/air, and He/air injection are presented The results seem to indicate that the simplified approach of this paper is surprisingly good for a gas such as CO_2, since a table for CO_2/air injection corresponding to Table 1 would differ very little Helium, on the other hand, behaves even more differently than one would expect from the fact that $\rho\mu$ for helium differs from $\rho\mu$ for air by about an order of magnitude For this reason the simplifications of $Le = Pr = Sc = 1$ may not adequately represent the case of a monatomic or light gas injected into the boundary layer

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Stratification in a Pressurized Container with Sidewall Heating

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A method to determine the temperature profile due to stratification in a pressurized tank containing a liquid and subjected to sidewall (aerodynamic) heating is described It is shown that a straightforward empirical approach yields satisfactory correlation with the available data

Nomenclature

A = tank wall heated area, ft²
c = constant, ft²
C_p = liquid specific heat, Btu/lb- R
D = tank diameter, ft
I = ratio of heat absorbed in stratified layer to heat input at sidewall
Q = heat transfer rate through wall, Btu/sec
T = temperature, R
t = time, sec
y = distance below surface, ft
ρ = density, lb/ft³

Subscripts

s = saturation
b = initial bulk temperature

Introduction

In Ref 1, the importance of determining the degree of stratification occurring in a cryogenic propellant tank due to aerodynamic heating was discussed A method was developed to predict the depth of the stratified layer and the maximum temperature; however, no attempt was made to determine the temperature gradient in the stratified layer By use of the data obtained by the authors in Ref 1, as well as other data, it will be shown that a straightforward empirical approach yields satisfactory correlation with stratification temperature data obtained in tests

Discussion

The heat input from the wall of a tank will tend to warm the liquid near the wall and will cause a convective current to move up toward the surface When the warm liquid reaches the vicinity of the surface, it will spread toward the center of the tank and form a mechanically stable configuration with the warmest liquid at the surface Further, since the rising liquid leaves its energy source and, in effect, turns a corner, the stratified layer at the surface will have a different temperature profile than that which existed in the boundary layer at the wall

If the heat flux at the wall is low, the boundary layer that forms will move up the wall in the manner described in free convection theory If the heat flux is high, boiling will take place and, in most cases, the boiling will be in the nucleate range Since the tank is pressurized, the bulk liquid is effectively in a subcooled condition; the bubbles growing at the wall will collapse in the cooler boundary layer moving up the wall The net result is that the heat input at the wall produces a thin boundary layer at the wall which subsequently deposits hot liquid at the surface (Fig 1)

For a well-insulated tank, in which most of the heat input travels down from the top, it has been found[2] that the temperature profile is similar in shape to that obtained for a

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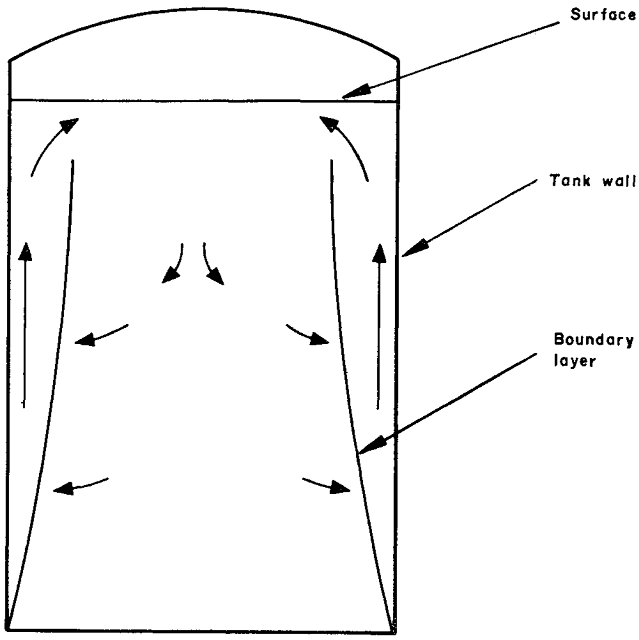

Fig 1 Convective transfer and stratification

semi-infinite slab whose surface temperature has been instantaneously raised to the saturation temperature The resulting temperature profile is convex upward (Fig 2) The basic mechanism of heat transfer for this case is conduction

For the case of a sidewall convective layer depositing warm liquid at the surface it would be expected that the profile would be different, and the temperature gradient at the surface would be less steep, i e , the warm liquid would "pile up" at the surface Some data obtained with a relatively small sidewall heat leak and a significant heat leak down from the top of the container[3] showed a convex-upward profile initially, with concave-upward at a latter period when the effect of the sidewall heat input and resulting convection became significant For a pressurized liquid-nitrogen tank subjected to aerodynamic heating, a concave-upward profile was always found [1 4 9] It was also found[4] that the temperature profile in the stratified layer was similar in shape to a Gaussian probability distribution and could be written as

$$T - T_b = (T_s - T_b)\, e^{-cy^2} \qquad (1)$$

By use of Eq (1), a method will be developed to determine the temperature profile as a function of various system parameters

For a cylindrical tank, the following heat balance can be written over an incremental height:

$$\int_0^y \frac{Q}{A} It\pi D dy = \int_0^y \frac{\pi D^2}{4} \rho C_P (T - T_b) dy \qquad (2)$$

The left-hand integral is simply the total heat input to the liquid times the fraction absorbed in the stratified layer If ρ and C_p are strong functions of temperature, their functions can be substituted in the equation For the purposes here, however, the average values of ρ and C_p are assumed to be satisfactory, and Eq (1) can be substituted into Eq (2) to yield

$$QIt = \frac{\pi D^2}{4} C_p (T_s - T_b) \int_0^y e^{-y^2} dy \qquad (3)$$

The integral in Eq (3) is the probability integral and has a maximum value (at y equal to infinity) of $\frac{1}{2}(\pi/c)^{1/2}$ The integral reaches a value of 99% of the value at infinity for $c^{1/2}y$ greater than 1 83 For other values of y, the integral must be found from tables [5] The integral converges very quickly to the value at infinity, and, for the usual case where stratification does not extend to the bottom of the tank, the value of the integral of Eq (3) for $y = \infty$ can be used with little error The value of the constant in Eq (1) is thus

$$c = \pi \left[\frac{\pi D^2}{4} \frac{\rho C_p (T_s - T_b)}{2QIt} \right]^2 \qquad (4)$$

The basic engineering problems in using Eqs (1) and (4) are determining the fraction of sidewall heat input that is absorbed in the stratified layer (I) and determining the surface temperature (T_s)

The sidewall heat input can be accounted for in four areas: 1) the stratified layer, 2) the bulk liquid, 3) the boundary layer along the wall, and 4) evaporated propellant It has been found that, for smooth-wall tanks with heat flux similar to that experienced due to aerodynamic heating, the total sidewall heat flux appears in the stratified layer [3 4 6 9] For any specific case, the amount of energy (heat) in the boundary layer can be found by integrating the temperature and velocity profile in the boundary layer The amount of

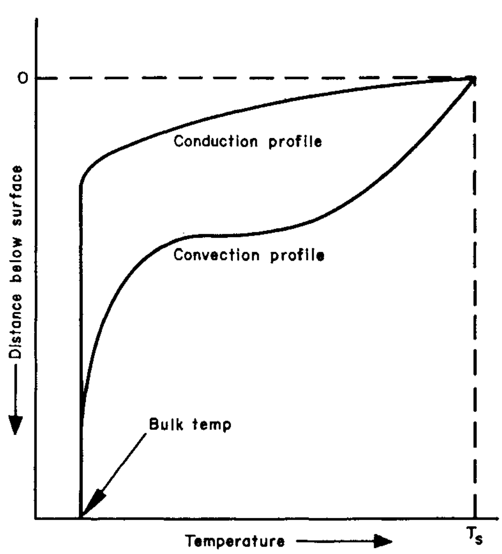

Fig 2 Stratification temperature profiles

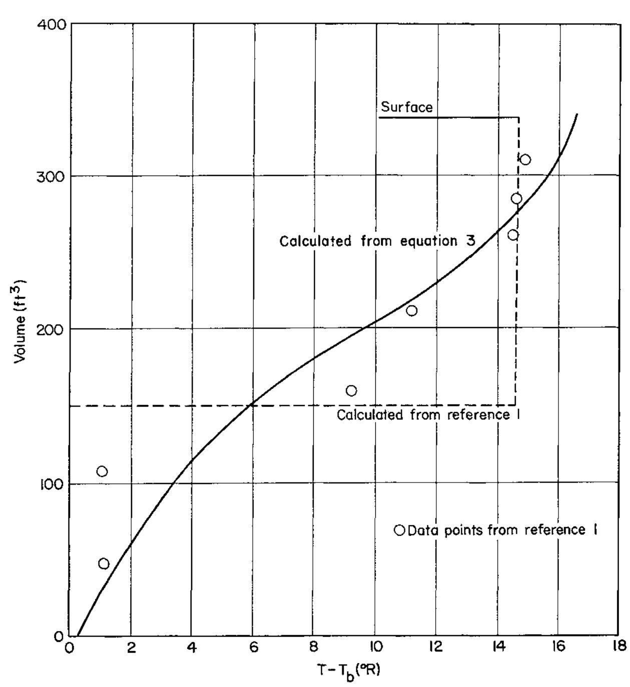

Fig 3 Stratification in a liquid nitrogen tank

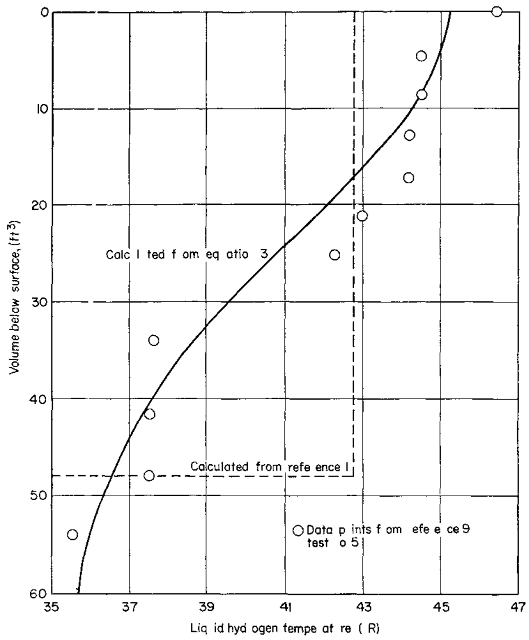

Fig 4 Stıatification in a liquid hydrogen tank

evaporated (or condensed) propellant can be found using a method outline by Knuth [7] For smooth-wall tanks, there is insignificant mixing between the boundary layer and the bulk liquid since the convective layer at the wall acts as a channel directing flow to the surface Thus, for most cases with smooth-wall tanks, the assumption that I is equal to one is satisfactory

If the tank contains horizontal structural members or anti-slosh baffles attached to the wall, the boundary layer growing at the wall will be diverted into the bulk liquid For this nonsmooth wall configuration, the heat absorbed by the bulk liquid cannot be calculated with any degree of accuracy At present there are no published theories or experimental data on the effect of a baffle on a free convection stream One approximation that can be used is to assume that the flow around the edge of a baffle is flow from an infinite line source and use the data obtained by Rouse[8] on air to calculate the spreading effect of the baffle

For the case of an ullage pressurized with vapor derived from the liquid phase, the surface temperature is for all practical purposes the saturation temperature at the existing ullage pressure For other cases the surface temperature will be between the bulk temperature and the saturation temperature If the surface temperature cannot be calculated, little error is usually introduced by assuming that the surface temperature is saturated at the ullage pressure

Figure 3 compares the calculated and measured temperature profiles in a liquid-nitrogen tank subjected to simulated aerodynamic heating It can be seen that good agreement is obtained even though the ullage was pressurized with helium gas and the surface temperature was less than the saturated temperature at the given ullage pressure It was assumed that all the heat input was absorbed by the stratified layer ($I = 1$) Comparison with data obtained for LH_2 also shows good agreement with calculated results (Fig 4) Also plotted on these figures are the calculated results using the method developed in Ref 1

The effect of vehicle slosh, even at the resonant frequency, was found to be minor [9] In addition, data indicate that the temperature profile did not change appreciably during tank drainage, i e , converting the temperature profile to an outlet temperature as a function of time showed good agreement with the previously measured profile in the tank

It appears that the suggested temperature profile [Eq (1)] using the calculated constant [Eq (4)] yields results that agree satisfactorily with test data obtained with various liquids For tanks with structural members or anti-slosh baffles that interfere with the boundary layer convective flow, the calculation of the amount of stratification is much more difficult However, the use of baffles suggests that the degree of stratification can be controlled by proper design of horizontal structural members which will introduce mixing of the boundary layer with the bulk liquid Further experimentation is needed in the area of nonsmooth walls to evaluate the effect such members have on thermal stratification

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Determination of the Mass of Gas in a Rapidly Discharging Vessel

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Nomenclature

a = speed of sound
A = cross-sectional area
k = ratio of specific heats
L = length of vessel
m = mass
P = pressure
t = time
u = flow velocity
V = volume
x = distance along axis of vessel
ρ = density
ϕ = A_e/A_t

Subscripts

e = conditions at minimum cross sectional area of constriction
f = conditions at minimum cross sectional area of flow stream
i = conditions just ahead of constriction
0 = initial conditions in the vessel
s = surroundings
t = vessel

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